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Original Article

Introducing a Bi-Level Linear Programming Model to Reduce Patient Payment and Increase Hospital Income Simultaneously

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Abstract

Background: The cost of health care is a large part of every household's budget. On the other hand, as an economic entity, the hospital is constantly faced with different aspects of cost and revenue. So, we are dealing with conflicting objectives.

Objectives: The main purpose of the research is to help financial management in a specialty hospital. This article provides part of operational research under bi-level optimization for hospital managers to provide targeted financial planning. The method is based on the fact that the objective is to maximize the hospital income on one level, and on the other level, the objective is to reduce the patient's payment.

Methods: The hierarchical and decentralized optimization problem is written as a bi-level model that minimizes patient costs and maximizes hospital revenues, which is an NP-Hard problem. The optimal solution to this problem is obtained using a genetic algorithm. Then, the hospital's performance is evaluated by the Pabon Lasso diagram. It is shown that the use of this model has a significant effect on the hospital's performance.

Results: Implementation of this model in the studied hospital shows that patient payment costs decreased and hospital income increased (reaching equilibrium point).

Conclusion: Hospital performance after model implementation was evaluated by the Pabon Lasso diagram and showed that it has an effective role in hospital performance.

Keywords: Bi-level Optimization Model, Health Care Costs, Hospitals, Genetic Algorithm, Pabon Lasso

1. Background

Today, the main challenge in health management is to reduce the costs paid by patients and increase hospital income. It means that the patient tries to pay as little as possible; on the other hand, the hospital tries to increase the income, which is a conflict of objectives.

With the high cost of health care, nearly 44 million households (more than 150 million people) worldwide face staggering costs each year.[1] Even out-of-pocket health insurance seems to increase the risk of poverty.[2] In low-income countries, lack of insurance or poor coverage and insufficient social support have led to out-of-pocket payments for households.[3]

Hospitals are one of the health care system components whose function in coordination with other factors leads to the community's health.[4] In fact, hospitals have a key role in providing health services, and because of this importance, they have a great impact on the efficiency of the health system. Among the indicators for measuring the efficiency of hospitals, the length of stay of patients is an important indicator and one of the simplest indicators of hospital activity that is widely considered today.[5] This criterion is used for different purposes: hospital care management, quality control, appropriateness of hospital services, and hospital planning.[6] The length of stay of patients is a function of the variables of medical services, including the availability of hospital beds, payment methods, and hospital discharge policies, as well as the variables of demand for medical services include disease severity, direct and indirect costs of the patient and concomitant diseases. Among the factors affecting the length of stay of patients, the method of payment and the use of basic and supplementary health insurance can have a significant impact on the length of stay, as in recent years, many studies have been conducted on the impact of payment methods on health behaviors, especially on length of stay in developing countries.[7]

2. Objectives

We want to design a model to reduce patient costs. To do this, we offer a model that reduces bed costs, surgical costs, and paraclinical costs.

On the other hand, we know that the hospital faces different aspects of cost and income as an economic unit. Therefore, hospitals are always reluctant to use economic analysis to increase efficiency and productivity. So, our objective is to increase hospital revenue as well.

3. Methods

This study has been used the bi-level programming model. The bi-level linear programming problem is a nested optimization model consisting of two decision-maker problems: the leader (upper level) and the follower (lower-level); the former controls the $x \in X \subset \mathbb{R}^n$ vector and the latter the $y \in \mathrm{Y} \subset \mathbb{R}^m$ vector.[8] The leader selects an x and tries to maximize $F(x,y)$ possibly under some constraints, and the follower, observing the leader's decision, selects a y and maximizes its objective function $f(x,y)$ for the given value x under certain constraints; it is worth noting that the leader influences the objective and the decision space of the follower. The bi-level linear programming problem can be written as follows:

$$\max_{x\in X} \quad \mathrm{F(x,y)} = c^1x + d^1y \tag{1}$$
$$s.t.$$
$$\max_{y\in Y} \quad \mathrm{f(x,y)} = c^2x + d^2y \tag{2}$$
$$s.t. \qquad Ax + By \le b$$

Where c^1 and $c^2 \in R^n$, d^1 and $d^2 \in R^m$, $b \in R^p$, $A \in R^{p\times n}$, and $B \in R^{p\times m}$; c^2x in the follower's objective function has a fixed value (for the leader's selected x) and can be omitted.

We now describe the genetic algorithm (GA) for solving the bi-level linear programming problem proposed by Wang et al.[9] One of the advantages of this algorithm is that not only the initial population but also the chromosomes produced by genetic operators are feasible, which reduces the search space and reduces the difficulty of finding feasible points. Also, replacing the best children with parents in operator processes increases the algorithm's efficiency. Floating coding has been adopted for this algorithm because it is closer to the problem space and has the advantage of faster convergence and higher computational accuracy in executions than binary coding.

The steps of Wang et al.'s GA to solve the bi-level linear programming problem are as follows:

Step 1. (Initializing) In this step, the algorithm's parameters are set, and the initial population $p(0)$ is generated. The population size M denotes the number of chromosomes in the population. Crossover probability p_c indicates how often crossover will be performed, and mutation probability p_m indicates how often parts of the chromosome will be mutated. Finally, the maximal iterated generation T is given to be used as the termination conditions of the algorithm. Then, the initial population $p(0)$ is generated according to the following procedure:

The upper-level decision variable x is randomly selected p, then for this fixed $x \in p$, the lower level programming (2) is solved, and let $y(x)$ denote the optimal solution. The point $(x, y(x))$ as a chromosome of the initial population is the feasible bi-level linear programming problem. Then the fitness value of the chromosome $(x, y(x))$ is defined as follows:

$$F^{/}\left(x, y(x)\right) = F\left(x, y(x)\right) - F_{\min} \tag{3}$$

Where $F_{\min} = \min_{(x,y)\in S} F(x,y)$. After sufficient generating of such chromosomes, set the generation counter $t=0$, then go to step 2.

Step 2. (Selection) The chromosomes of the population P (t+1) are selected from the population $P(t)$ according to their fitness by the roulette wheel. After selecting the population P(t+1), go to step 3.

Step 3. (Crossover) The crossover operator is carried on the population P (t+1) in this step. For each chromosome in the population P(t+1), a random number α ϵ [0,1] is generated. If $\alpha < p_c$ then, this chromosome is selected to crossover. The procedure of crossover is as follows:

Assuming two chromosomes $(x_1, y(x_1))$ and $(x_2, y(x_2))$ are selected from the population as the parents. Then,

$$x_{01} = \beta x_1 + (1-\beta)x_2,\; x_{02} = (1-\beta)x_1 + \beta x_2$$

Where β ϵ [0,1] is randomly generated and guaranteed $x_{01}, x_{02,} x_{04}, x_{04} \in p$. Then the lower-level programming (2) is solved fog x_{01}, x_{02}; thus, we can get two offspring $(x_{01}, y(x_{01}))$ $(x_{02}, y(x_{02}))$, which are all feasible for bi-level linear programming problems If the offspring are better than the parents, they replace the parents. Then go to step 4.

Step 4. (Mutation) The mutation operator is carried on the populationin P (t+1) this step. For each chromosome in the population P (t+1), a random number α ϵ [0,1] is generated. If $\alpha < p_m$, then this chromosome is selected to mutate. Assuming one chromosome $(x, y(x))$ is selected from the population as the parent. Then the integer l ϵ $[1,n_1]$ is randomly generated, and the l^{th} component of the chromosome is mutated according to the following procedure:

$$x_0(l) = \begin{cases} x(l) + \beta(x_{ub}(l) - x(l)), & if\ \ random(0,1) = 0 \\ x(l) - \beta(x(l) - x_{lb}(l)), & if\ \ random(0,1) = 1 \end{cases} \tag{4}$$

Where $x_{ub}(l)$, $x_{lb}(l)$ denote the variable's upper and lower bound x, respectively. $\beta \in [0,1]$ is randomly generated and guaranteed $x_0 \in p$, and other components of x_0 are the same as the corresponding components x, then the lower-level programming (2) is solved for x_0. Thus, the offspring $(x_0, y(x_0))$ is obtained, which is feasible for a bi-level linear programming problem. If the offspring is better than its parent, it replaces the parent. Then go to step 5.

Step 5. (Termination) The algorithm terminates when the generation t is greater than the maximal iterated generation T.

The best generated solution, which has been recorded in all iterations, is reported as the optimum for bi-level linear programming problems by the proposed GA algorithm. Otherwise $t=t+1$, and go to step 2.

The proposed GA and the pseudo-code are shown in Figure 1 and Figure 2.

Studies show different indicators for measuring hospital performance, the most important and the most applicable being bed occupancy rate, bed turnover rate, and the average length of stay the patient is in the hospital. Pabon Lasso's drawing model is one of the most useful models capable of simultaneously comparing hospital performance indices. Pabon Lasso first introduced a universal model for Columbia Hospital in 1986.[10] The Pabon Lasso model is one of the performance measurement methods that shows the appropriate utilization of available hospital resources. Paben Lasso's model bed occupancy rate, bed turnover rate, and the average length of stay the patient of stay in a way that allows for a better interpretation of their meaning. In the Paben Lasso diagram (Figure 3), the data on the occupancy rate of each hospital is in the X-axis coordinate axis and the bed rotation frequency data in the vertical Y-axis, and the mean days of hospitalization in the Z-axis.[11,12] A three-dimensional graph is created that is not easy to draw in two-dimensional space. Accordingly, the mathematical relationship between these three indices is the average linear days of hospitalization, from the coordinate origin to the hospital location in each of the four regions, and continues to the opposite side. Then it increases from left to right and from bottom to top.[13]

Now the mathematical model of bi-level linear programming is presented.

Indices, variables, and parameters of patient cost reduction are considered as follows:

Indicators:

$w, t =$ General indicator

Variables:

$X_{wt} =$ number of day

$Y_{wt} =$ Rial coefficient by insurance

$N_{wt} =$ Number of service

Parameters:

$C_{wt} =$ Cost required bed

$A_{wt} =$ Cost of surgical rial value

$K_{wt} =$ Worth serving

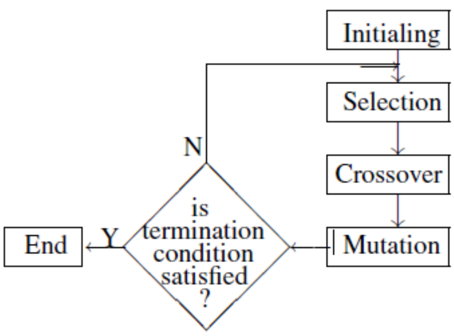

Figure 1. Flow Chart of Proposed GA Algorithm.

```
begin
  initialize;
  generating;
  t = 0;
  repeat
    evaluate P(t);
    select P(t + 1) from P(t);
    crossover and mutate P(t + 1);
    t = t + 1;
  until (termination condition);
end
```

Figure 2. Pseudo-code for the Algorithm.

Region 2: -low occupancy -high turnover -short stay	**Region 3:** -high occupancy -high turnover -short stay
Region 1: -low occupancy -low turnover -long-stay	**Region 4:** -high occupancy -low turnover -long-stay

Figure 3. Four Areas of the Pabon Lasso Diagram.

$H =$ Maximum patient pay for bed

$G =$ Maximum patient pay forSurgery

$R =$ Maximum patient pay for Paraclinical

Indices, variables, and parameters of hospital revenue increase are considered as follows:

Indicators:

$w_i, t_i =$ High − income surgical index

$w_j, t_j =$ low − income surgical index

Variables:

$D'_1 =$ Bed occupancy factor for high-income surgery

$D'_2 =$ Patient turnover coefficient for high-income surgery

$D'_3 =$ Paraclinical Services Coefficient for high-income surgery

$D''_1 =$ Bed occupancy factor for low-income surgery

$D''_2 =$ Patient turnover coefficient for low-income surgery

$D_3^{''}$ = Paraclinical Services Coefficient for low-income surgery

Parameters:

$X_{w_i t_i}$ = Number of days for high-income surgery

$Y_{w_i t_i}$ = Rial coefficient on insurance for high-income surgery

$N_{w_i t_i}$ = Number of services for high-income surgery

$X_{w_j t_j}$ = Number of days for low-income surgery

$Y_{w_j t_j}$ = Rial coefficient on insurance for low-income surgery

$N_{w_j t_j}$ = Number of services for low-income surgery

$C_{w_i t_i}$ = bed Cost Required for high-income surgery

$A_{w_i t_i}$ = The cost of the surgical rial for high-income surgery

$K_{w_i t_i}$ = Worth serving for high-income surgery

$C_{w_j t_j}$ = bed Cost Required for low-income surgery

$A_{w_j t_j}$ = The cost of the surgical rial for low-income surgery

$K_{w_j t_j}$ = Worth serving for low-income surgery

Q = Maximum hospital income for high surgery bed

E = Minimum hospital income for low surgery bed

T = Maximum hospital income for high surgery

U = Minimum hospital income for low surgery

O = Maximum hospital income for high surgery Paraclinical

V = Minimum hospital income for low surgery Paraclinical

Then the model is as (5).

We solve the bi-level linear programming model (5) by the proposed algorithm. This study examines 40 patients under four different insurances in a specialty hospital. These patients have been referred for ten different surgeries: cesarean section, appendicitis, cholecystectomy, hemorrhoids, herniorrhaphy, septoplasty, laminectomy, varicocele, tonsillectomy, plaque removal.

On the other hand, we divide high and low-income surgeries as follows.

$$
\begin{aligned}
&\min\Big(\sum_w\sum_t C_{wt}X_{wt} + \sum_w\sum_t A_{wt}Y_{wt} + \sum_w\sum_t K_{wt}N_{wt}\Big)\\
&\text{s.t.}\quad \sum_w\sum_t C_{wt}X_{wt} \le H\\
&\qquad \sum_w\sum_t A_{wt}Y_{wt} \le G\\
&\qquad \sum_w\sum_t K_{wt}N_{wt} \le R\\
&\qquad X_{wt} \ge 0,\ Y_{wt} \ge 0,\ N_{wt} \ge 0
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&\max\quad D_1^{'}\Big(\sum_{w_i}\sum_{t_i} C_{w_i t_i}X_{w_i t_i}\Big) + D_1^{''}\Big(\sum_{w_j}\sum_{t_j} C_{w_j t_j}X_{w_j t_j}\Big) + D_2^{'}\Big(\sum_{w_i}\sum_{t_i} A_{w_i t_i}Y_{w_i t_i}\Big)\\
&+D_2^{''}\Big(\sum_{w_j}\sum_{t_j} A_{w_j t_j}Y_{w_j t_j}\Big) + D_3^{'}\Big(\sum_{w_i}\sum_{t_i} K_{w_i t_i}N_{w_i t_i}\Big) + D_3^{''}\Big(\sum_{w_j}\sum_{t_j} K_{w_j t_j}N_{w_j t_j}\Big)\\
&\text{s.t.}\quad D_1^{'}\Big(\sum_{w_i}\sum_{t_i} C_{w_i t_i}X_{w_i t_i}\Big) \le Q\\
&\qquad D_1^{''}\Big(\sum_{w_j}\sum_{t_j} Cw_j t_j X_{w_j t_j}\Big) \ge E\\
&\qquad D_2^{'}\Big(\sum_{w_i}\sum_{t_i} A_{w_i t_i}Y_{w_i t_i}\Big) \le T\\
&\qquad D_2^{''}\Big(\sum_{w_j}\sum_{t_j} A_{w_j t_j}Y_{Y_{w_i t_i}}\Big) \ge U\\
&\qquad D_3^{'}\Big(\sum_{w_i}\sum_{t_i} K_{w_i t_i}N_{w_i t_i}\Big) \le O\\
&\qquad D_3^{''}\Big(\sum_{w_j}\sum_{t_j} K_{w_j t_j}N_{w_j t_j}\Big) \ge V\\
&D_i^{'} \ge 0, D_j^{''} \ge 0 : i = 1,2,3.\ j = 1,2,3.
\end{aligned}
$$

High-income surgeries: Cesarean section, appendicitis, cholecystectomy, laminectomy, and plaque removal.

Low-income surgeries: Hemorrhoids, herniorrhaphy, septoplasty, varicocele, and tonsillectomy.

The parameters are set as in Table 1.

4. Results

The convergence diagrams of the GA are as follows.

In Figure 4 and Figure 5, fitness represents the value of the leader function concerning the values of the best child variables in each generation. In Figure 6 and Figure 7, fitness represents the value of the follower function concerning the values of the best child variables in each generation.

In this study, to obtain the required information using the expressed model, we obtained the required three performance indices using the computational formulas as shown in Table 2.

Using Excel program and the Pabon Lasso chart, the efficiency and performance of the hospital are as follows:

As shown in Figure 8, all 10 surgeries examined by the hospital are in the third region, indicating that the model is performing well.

Table 1. Parameter Setting for Model (2) Solving Using GA

Leader	Follower
Popsize: 300	Popsize: 300
Crossrate: 0.06	Crossrate: 0.06
Mutrate: 0.06	Mutrate: 0.06
Generation: 100	Generation: 100

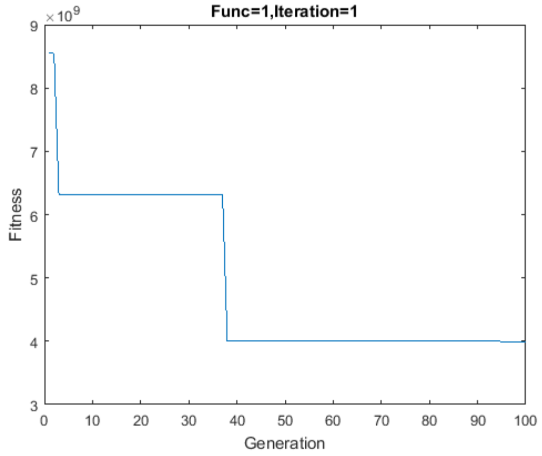

Figure 4. Leader, the First Iteration.

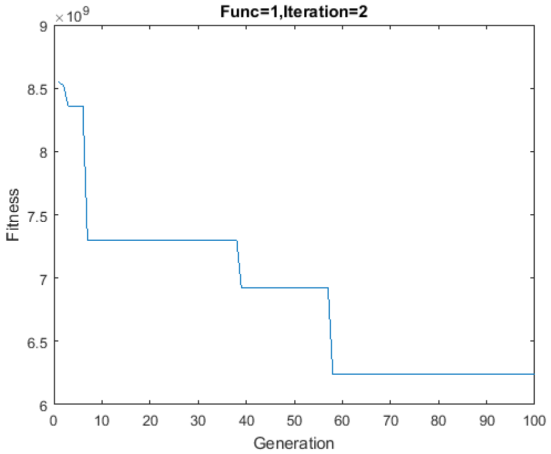

Figure 5. Leader, the Second Iteration.

5. Discussion

One of the most important challenges facing hospital management and the healthcare industry is revenue and cost management, which must meet patients' expectations. While expecting to receive good and acceptable services, patients tend to reduce their treatment costs as much as possible. On the other hand, the hospital management is trying to earn more money due to increased hospital costs. Research background and library studies have shown that the bi-level model can solve this problem. By mathematical modeling, the patient's cost can be reduced while reducing the length of hospital stay.

On the other hand, increasing the allowable bed rotation time increased the hospital income. By selecting repetitive surgeries and implementing a mathematical model, Since the obtained mathematical problem is the NP-Hard, the bi-level problem of optimizing patients' costs and hospital income was solved by using the genetic algorithm. According to Table 2, the findings show that the highest percentage of bed occupancy is related to appendicitis patients, and the lowest value of this index is related to hemorrhoid patients. The highest percentage of increase in the bed occupancy cycle is related to plaque removal patients, and the lowest value of this index is related to septoplasty patients. Simultaneously comparing the above indicators using Figure 8, All ten surgeries are examined in region 3, which shows that the proposed model has good performance. Using the information obtained from the analysis of efficiency indicators in hospitals, better allocation of resources and optimal productivity of existing beds can be achieved. The results show that in repetitive surgeries, the use of bi-level problems has reduced the cost of hospitalization by reducing the allowable time of hospitalization. At the same time, as increasing the permissible bed turnover, the hospital revenue will increase.

6. Conclusion

This article deals with the issue of health, considering that the main mission of the health system is to improve the level of health and respond to the needs of the people and society. These needs constantly change under economic, social, political, and environmental conditions. Health care costs make up the bulk of every household's cost basket. On the other hand, as an economic entity, the hospital is always faced with different aspects of cost and revenue. Therefore, we have designed a bi-level linear programming problem to reduce patient pay and increase hospital revenue simultaneously. Implementation of this model in the studied hospital according to Figures 4-7

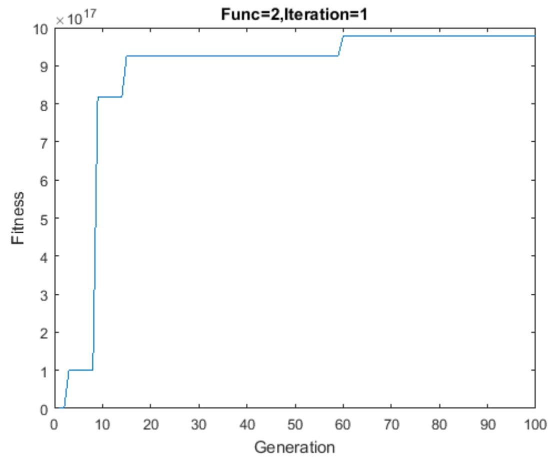

Figure 6. Follower, the First Iteration

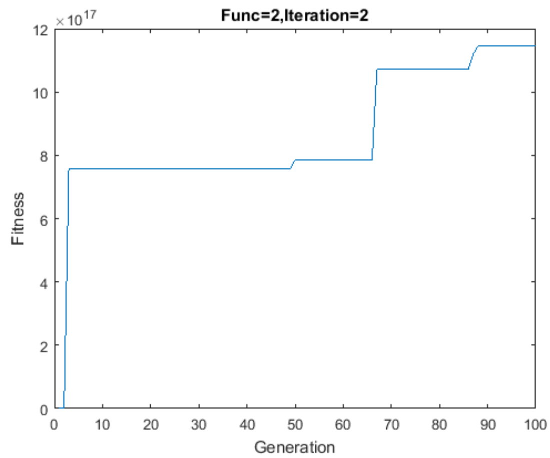

Figure 7. Follower, the Second Iteration

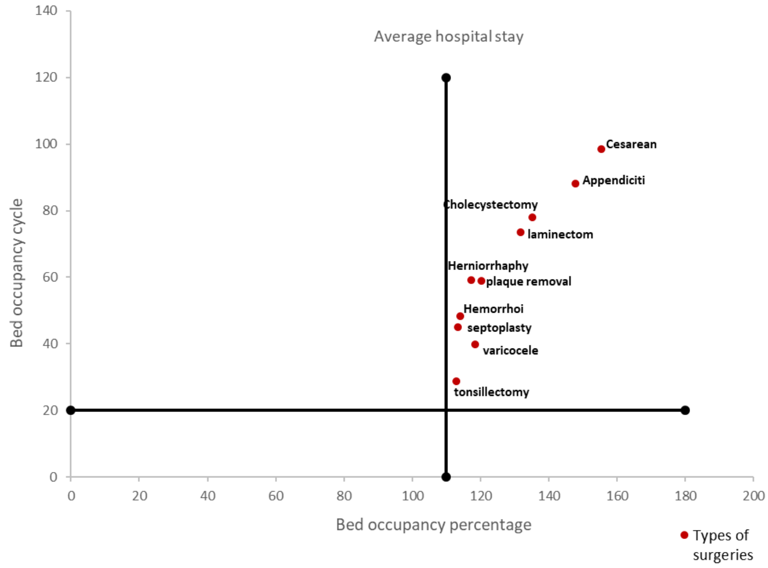

Figure 8. Location of the Studied Hospital Based on Pabon Lasso Model.

Table 2. Performance Indicators Separate the Surgeries under Study

Percentage Increase Types of Surgeries	Percentage Increase of Bed Occupancy	Percentage Increase Bed Occupancy Cycle	Percentage Increase Average Hospital Stay
Cesarean	0.516	0.498	0.22
Appendicitis	0.947	0.288	0.15
Cholecystectomy	0.234	0.178	0.21
Hemorrhoids	0.114	0.448	0.27
Herniorrhaphy	0.317	0.259	0.30
Septoplasty	0.314	0.145	0.72
Laminectomy	0.713	0.673	0.11
Varicocele	0.411	0.939	0.92
Tonsillectomy	0.812	0.928	0.65
Plaque removal	0.120	0.958	0.17

Research Highlights

What Is Already Known?

The main challenge in health management is to reduce the costs paid by patients and increase hospital income. It means that the patient tries to pay as little as possible. However, the hospital tries to increase its income on the other hand.

What Does This Study Add?

By designing and solving a mathematical model in game theory in a competitive environment, patients' costs are reduced, and hospital revenues are increased.

shows that patient payment costs decreased and hospital income increased (reaching equilibrium point). Hospital performance after model implementation was evaluated by the Pabon Lasson diagram and shows that it has an influential role in hospital performance. As shown in Figure 8, it was able to place different hospital surgeries simultaneously in the third area of the Pabon Lasso model.

Authors' Contributions

All authors contributed equally to the study.

Conflict of Interest Disclosures

The authors declare that they have no conflicts of interest.

Ethical Approval

Not applicable.

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